IFireSS 2017 – 2nd International Fire Safety Symposium
Naples, Italy, June 7-9, 2017

ON THE EXTENSION OF A PLASTIC-DAMAGE MODEL TO HIGH TEMPERATURE AND FIRE



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ABSTRACT

Concrete modelling at high temperature is not an easy task due to its highly nonlinear behaviour. When simple members are at issue, numerical modelling can take advantage of uniaxial constitutive laws. On the contrary, when walls and tunnel linings have to be studied, finite elements must be used, thus requiring the multiaxial constitutive behaviour to be properly defined.

The aim of this paper is to show the calibration of the main parameters of a plastic-damage model via inverse analysis, on the basis of experimental data available in the literature, regarding tests under multiaxial state of stress in both hot and residual conditions. The results indicate that there is a strong dependence on temperature of the ratio between biaxial and uniaxial compressive strength in hot conditions, while it is definitely less evident in residual conditions. As regards the variation of the shape of the failure surface with temperature, it appears to be of secondary importance.

1 INTRODUCTION

Concrete modelling at high temperature is not an easy task due to the highly nonlinear material and structural behaviour. When irrecoverable deformation components at the material level are taken into account, it is often required to perform plastic analyses and, consequently, the use of advanced constitutive models implemented in finite element codes becomes mandatory.

When beams and columns are at issue, numerical modelling via beam finite elements [1] can be carried out by using the uniaxial constitutive laws, such as those provided in the pertinent standards [2]. On the other hand, when more complex structural members (such as ceilings, walls and tunnel linings) are considered, 2D or 3D finite elements analyses must be carried out, thus requiring the definition of concrete behaviour for multiaxial states of stresses. Within this context, a very popular model is the Concrete Damaged Plasticity (CDP) model [3, 4] implemented in the ABAQUS

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commercial software and originally set-up for modelling concrete behaviour under complex load cycles, featuring loading and unloading, development of irreversible deformations, as well as the ensuing degradation of the elastic properties.

In particular, in addition to the uniaxial mechanical properties (modulus of elasticity, compressive and tensile strength), in the CDP model two parameters must be defined in order to define the failure surface: the ratio between biaxial and uniaxial compressive strength ($f_{c,b}/f_{c,1}$), and the ratio between the second stress invariant on the tension meridian and the second stress invariant on the compression meridian (k_c).

The first parameter defines the extension of the failure surface in the Haigh-Westergaard space, and is relevant for states of biaxial-triaxial compression, whereas the second parameter is related to the shape of the failure surface on any deviatoric plane, and is thus more relevant for states of combined tension and compression.

A sound calibration of the aforementioned parameters is only possible by considering the scanty results on the behaviour of concrete under multiaxial states of stress. Some of the most interesting and well-documented are briefly recalled in the following.

Kordina et al. [5] carried out biaxial compression tests on normal-strength concrete specimens, with a cube strength of 41 MPa. The tests were carried out in steady state and transient temperature conditions (hot tests). For each thermal level, the results of the tests were interpreted with a failure envelope in the plane of principal stresses σ_1 and σ_2 ($\sigma_3 = 0$ for biaxial state of stress). The strength under biaxial compressive stress turned out to be higher than the strength under uniaxial compression. The relative increase of strength under biaxial stress is more significant at higher temperatures than at ambient temperature. Moreover, the failure temperature of the specimens under biaxial stresses is higher than that of specimens in uniaxial compression.

Thienel and Rostasy [6] studied the mechanical behaviour of normal-strength concrete subjected to high temperature (hot conditions) and biaxial stress. Tests were performed on unsealed concrete and mortar incorporating quarzitic aggregate. The results concerning the biaxial compressive strength at different temperatures are reported for two different concrete mixes with an initial uniaxial compressive strength of 45 and 37 MPa respectively. The temperature-induced decrease of strength is affected by the composition of concrete in the entire range of biaxial compressive stress. The maximum aggregate size has a significant influence, while aggregate and w/c ratio are found to play a minor role. Moreover, strength values are affected by concrete composition up to a temperature of 450°C. At this temperature, the decomposition of portlandite begins and significantly alters the structure of the cement paste. After the decomposition of portlandite, the aforementioned tendencies change or become more pronounced.

Ehm and Schneider [7] show that small load levels along the second axis significantly alter the mechanical properties of concrete at high temperature (hot conditions). Test results indicate an increase of ultimate biaxial strength at high temperature compared to uniaxial strength. Biaxial strength increase is comparatively larger at high temperatures with respect to residual conditions.

A series of tests were performed by He and Song [8] for characterizing the strength and deformation behaviour of two High-Strength High-Performance Concretes (HSHPC) at 7 different stress ratios, after the exposure to normal and high temperatures (residual conditions). The results showed that the uniaxial compressive strength of plain HSHPC after exposure to high temperatures does not decrease dramatically with the increase in temperature. The ratio of the biaxial to its uniaxial compressive strength depends on the stress ratios and brittleness-stiffness of HSHPC after exposure to different temperature levels. Moreover, the biaxial compressive strength is larger than its corresponding uniaxial compressive strength at all stress ratios for the same temperature level.

Taking advantage of the aforementioned test results, the aim of this work is to calibrate the main parameters of the CDP model via inverse analysis, on the basis of the experimental data available in the literature, regarding tests under multiaxial state of stress in hot [5-7,10] and residual [9,12] conditions.

2 CONCRETE DAMAGED PLASTICITY MODEL

2.1 Failure model

While well-established strength models for biaxially stressed concrete are available at room temperature, as for example [13], it is not the case for high temperatures. However, failure criterion for concrete during heating or in residual conditions can be based on that for normal temperature, if properly modified.

In a very general form, assuming tension stress to be positive, yield function for frictional materials like concrete and rock can be written as expressed in *Eq. (1)* [4].

$$\overline{F}(\sigma) - c = 0 \tag{1}$$

where c is cohesion and $\overline{F}$ is a scalar function of invariants of stress tensor. Cohesion c is represents material uniaxial compressive strength, which is temperature dependent. Yield function adopted in this work is the one proposed in [3] and [4] and implemented in ABAQUS software through Concrete Damaged Plasticity Model. In case of 3D stress states, it has the form of *Eq. (1)*.

$$\overline{F}(\sigma) = \frac{1}{1-\alpha}\left[\sqrt{3J_2} + \alpha I_1 + \beta\langle\sigma_{\max}\rangle - \gamma\langle-\sigma_{\max}\rangle\right] = f_c(T) \tag{2}$$

where < a > = a if a is positive and 0 otherwise.

I_1 and J_2 are the first and second stress deviator, respectively, as expressed in *Eqs. (3a,b)*; α and β are dimensionless constants depending on mechanical strength according to *Eqs. (4a,b)* and γ depends on the ratio K_c between second stress invariant on the tension meridian and second stress invariant on the compression meridian as expressed in *Eq. (4c)*

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 \tag{3a}$$

$$J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2\right] \tag{3b}$$

$$\alpha = \frac{(f_{bo}/f_{co}) - 1}{2(f_{bo}/f_{co}) - 1} \tag{4a}$$

$$\beta = (1-\alpha)(f_{co}/f_{to}) - (1+\alpha) \tag{4b}$$

$$\gamma = \frac{3(1-K_c)}{2K_c - 1} \tag{4c}$$

where f_{bo} and f_{co} are initial biaxial and uniaxial compressive yield stresses, respectively, and f_{to} is uniaxial tensile yield stress.

Parameter γ appears only in triaxial compression, since all principal stresses are negative (hence also σ_{max} is negative). On the other cases, the yield criterion has the form of *Eq. (5)*.

$$\overline{F}(\sigma) = \frac{1}{1-\alpha}\left[\sqrt{3J_2} + \alpha I_1 + \beta\langle\sigma_{\max}\rangle\right] \tag{5}$$

For biaxial compression (σ_{max} = 0) CDP matches the Drucker-Prager criterion (*Eq. (6)*).

$$\overline{F}(\sigma) = \frac{1}{1-\alpha}\left[\sqrt{3J_2} + \alpha I_1\right] \tag{6}$$

Then, the only parameter to be calibrated is α, by evaluating biaxial and uniaxial compressive strengths. Once α is known, β can be determined evaluating the ratio of initial uniaxial compressive and tensile yield stress.

In addition to uniaxial mechanical properties of concrete (compressive and tensile strength and modulus of elasticity) parameters α and β must be defined to completely describe the failure surface in biaxial stress states.

These parameters are evaluated for each temperature, so that the yield function represents experimental data as accurately as possible. In the following, the fitting procedure used for calibration of these two parameters will be explained.

Having the stresses measured in two principal planes and compressive strength at each test temperature, it is possible to calibrate the parameters α and β to fit the experimental data for each temperature. This will enable to study the variation of biaxial-to-uniaxial strength ratio with temperature. However, it should be kept in mind that the calibration of parameters to fit the experimentally measured strengths should include not only the main variable which is temperature, but also all the variables which have a significant influence on high temperature strength, such as concrete mix design, test conditions (hot or residual) etc. [7]. Moreover, experimental data should report the variation of compressive strength with temperature.

Experimentally measured data (σ_1, σ_2, and f_c) are available from several tests, in which concrete was biaxially stressed [5-7, 12]. Tests by He and Song [12] were performed in residual conditions, i.e. after exposure to high temperatures, while other in hot conditions. Compressive strength variation with temperature for each of the tests is reported in *Fig. 1*.

2.2 Fitting procedure

Let us assume one function f which relates unknown parameters X_1^*, X_2^*, …, X_m^* to the measurable quantities P_1, P_2, …, P_s as expressed by *Eq. (7)*.

$$f\left(X_1^*, X_2^*,..., X_m^*; P_1, P_2,..., P_s\right) = v_i \tag{7}$$

where v_i is the residual and function f is a failure criterion proposed by Lubliner [3].

In the present case, unknown parameters are α and β (and γ in triaxial stress state) while measurable quantities are two principal stresses (three in triaxial stress states) and initial compressive strength. To obtain the best fit curve, v_i should be minimized, as for example via least squares method.

System of two or three equations (depending if 2D or 3D stress state) is then linearized, assuming that the value of the function f is known at one point. It is therefore necessary to start by considering X_1^0, X_2^0, …, X_m^0 as approximate values of X_1^*, X_2^*, …, X_m^*.

This enables to redefine the unknowns, by adding the correction values x_1, x_2, …, x_m to the approximate values, as described by the set of *Eqs. (8)*.

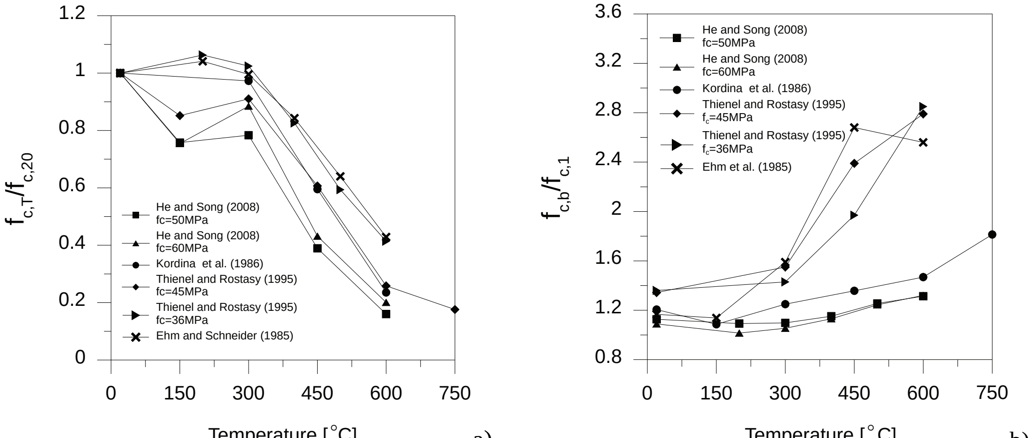


Fig. 1. Test results considered in the fitting procedure: a) normalized decay of the uniaxial compressive strength; and b) ratio between biaxial and uniaxial compressive strength as a function of temperature

$$X_1^* = X_1^{\ 0} + x_1$$
$$X_2^* = X_2^{\ 0} + x_2$$
$$...$$
$$X_m^* = X_m^{\ 0} + x_m \tag{8}$$

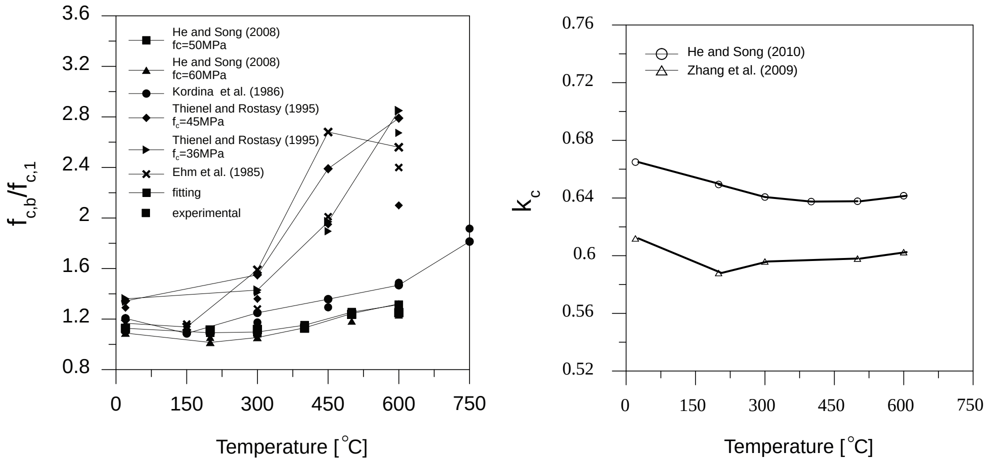
Developing Taylor series of a function *f, Eq. (9)* is derived.

$$f_i\left(X_1^*, X_2^*,..., X_m^*; P_1^i, P_2^{\ i},..., P_s^{\ i}\right)=$$
$$= f_i\left(X_1^0, X_2^{\ 0},..., X_m^{\ 0}; P_1^i, P_2^{\ i},..., P_s^{\ i}\right) + \left(\frac{\partial f}{\partial X_1}\right)_i^0 x_1 + \left(\frac{\partial f}{\partial X_2}\right)_i^0 x_2 + ... + \left(\frac{\partial f}{\partial X_m}\right)_i^0 x_m + R \tag{9}$$

Residual R contains all the higher order terms and in order to make the linearization of the equations possible, it has to be assumed that the values x_1, x_2, …, x_m are sufficiently small to make the value of R negligible.
Once the system of equations is solved and values of x_1, x_2, …, x_m are obtained, they are added to the approximate values of the unknowns. Nevertheless, this procedure is iterative and system of equations is being solved again for the new values of unknowns, until the solution which minimizes the residual is found.

3 RESULTS OF THE FITTING PROCEDURE

Calibrating the parameter α to fit the available experimental data, biaxial-to-uniaxial strength ratio can be determined together with its evolution with temperature. As expected, this ratio increases with the temperature as clearly shown in *Fig. 1*.
As for the fitting procedure results, it can be seen that the results are satisfactorily fitted and with a small error margin for experimental data from residual tests, where the dependence of ratio f_{bo}^T / f_{co}^T on temperature is less pronounced than in hot conditions and its value is not significantly increased with temperature.
As for the hot tests, better fit is obtained at lower temperatures, when the values of f_{bo}^T / f_{co}^T are still somewhat close to those at ambient conditions (α = 1.08-1.20). With significant increase in temperature and corresponding decay in uniaxial compressive strength, values of parameter α are sizably higher and the fitting accuracy is lower.



(a) (b)

Fig. 2. Results of the fitting procedure: comparison of (a) uniaxial-to-biaxial strength ratio and (b) K_c

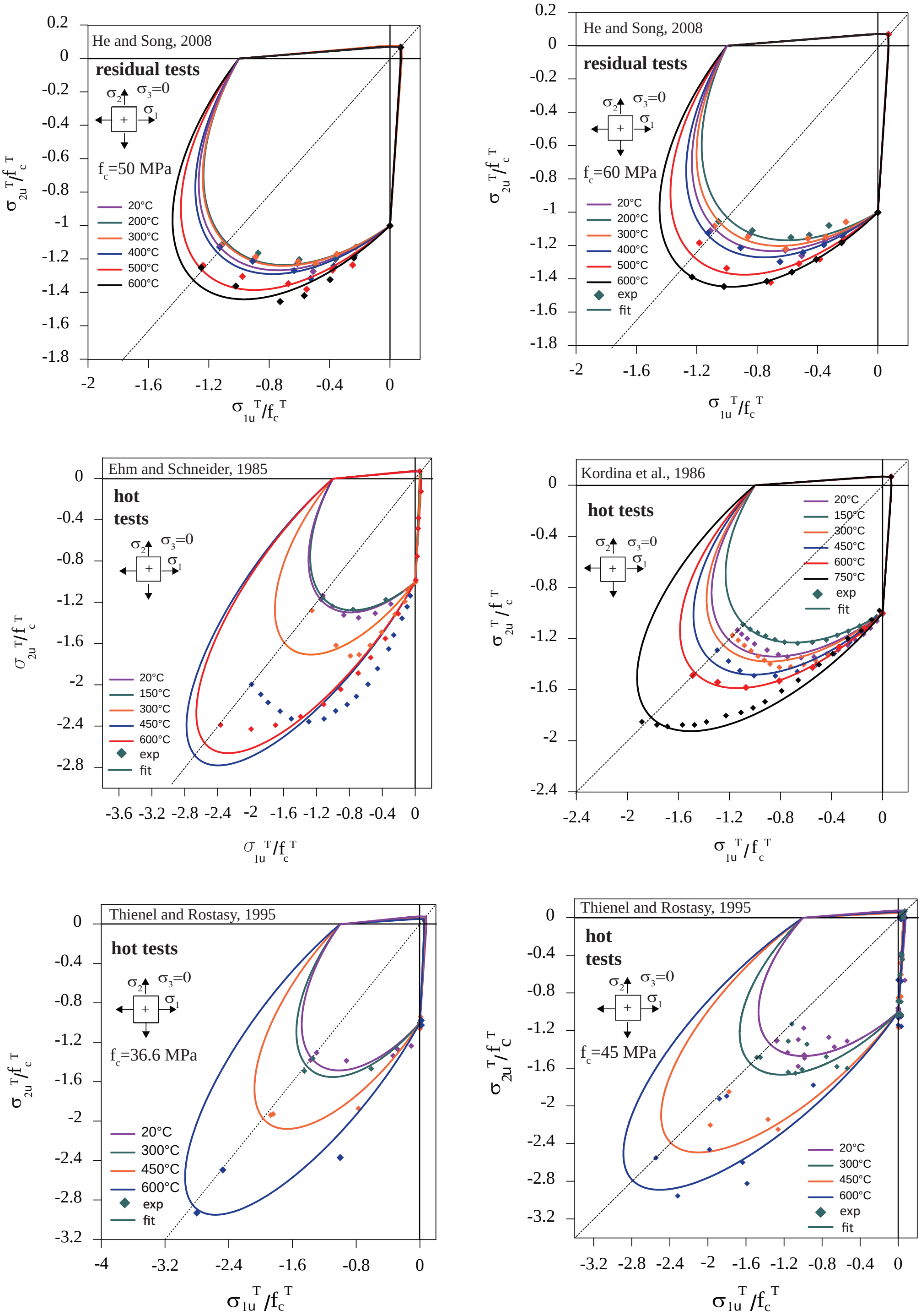


Fig. 3. Results of the fitting procedure: biaxial yield function

As for the triaxial tests [11,12], after performing fitting procedure on α, also the coefficient γ was calibrated for each test temperature. Its variation with temperature is far less pronounced (K_c = 0.660 at 20ºC and K_c = 0.644 at 600ºC).

4 CONCLUSIONS

In the present paper, the extension of a well-known plastic-damage model from room to high temperature is briefly discussed. The study focuses on Concrete Damaged Plasticity (CDP) model, since it is one of the most used approach for simulating concrete behaviour at high temperature.

Even though such criterion is rather widespread thank to its implementation on ABAQUS commercial software, systematic investigations on the variation of some concrete mechanical properties with temperature is still scanty. In particular, this is the case of the ratio f_{bo}^T / f_{co}^T between biaxial and uniaxial compressive strength, and of the ratio K_c between second stress invariant on the tension meridian and second stress invariant on the compression meridian (K_c being important in triaxial compression only).

The most common values of these two quantities at high temperature are f_{bo}^T / f_{co}^T = 1.16-1.20 and K_c = 0.667. A fitting procedure has been implemented to match the experimental results available in the literature on biaxial and triaxial compression tests, in hot and residual conditions, (namely at high temperature, and after heating and cooling down to room temperature, respectively).

The ratio f_{bo}^T / f_{co}^T in hot conditions showed to be strongly dependent on the temperature, going from values in the range 1.1-1.3 at room temperature to values in the range 2.0-3.0 at 600°C. In residual conditions the variation is less evident, since at 750°C the ratio is around 1.8.

It is also shown as the biaxial yield function (failure dominium) described in CDP model satisfactorily represents the experimental evidences, even though with a lower accuracy in hot conditions.

A regards K_c, experimental data are evaluable for residual conditions only. In such cases, K_c parameter proved to be far less temperature-sensitive than f_{bo}^T / f_{co}^T, remaining in the range 0.60-0.67 from 20 to 600°C, with only a slightly decrease with temperature. For K_c, is thus reasonable to keep a constant value (as for example the common value 0.667) for all the temperature range, at least in residual conditions.

Obviously, more experimental data must be analyzed for drawing more general conclusions. In the next future, further experimental fitting will be performed in order to get more robust results and parametric analyses will be carried out for quantifying the influence of these two parameters on the mechanical response of concrete during and after heating in case of multi-axial state of stress.

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